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Model Apparatus for Quantum Measurements

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We present a model system that behaves as a measurement apparatus for quantum systems should. The device is macroscopic, it interacts with the microscopic system to be measured, and the results of that interaction affect the macroscopic device in a macroscopic, irreversible way. Everything is treated quantum mechanically: the apparatus is defined in terms of its (many) coordinates, the Hamiltonian is given, and time evolution follows Schrödinger's equation. It is proposed that this model be itself used as a laboratory for testing ideas on the measurement process.

KEY WORDS: Quantum measurement theory; apparatus models.

1. INTRODUCTION

The devices by which quantum measurements are implemented tend to be complicated: Geiger counters, droplets in cloud chambers, cats. By contrast, the usual language for the analysis of such experiments is starkly abstract: Hilbert spaces for both measured coordinate and apparatus; sometimes the only recognition of the complexity of the apparatus is the use of capital letters for its state vectors. Few of the many papers of the last 60 years on quantum measurement theory have attempted to model the measurement process as it occurs within the apparatus, that is, how the microscopic signal of the measured object is promoted to macroscopic consequence. To us it seems that such a project is part of the general program of statistical mechanics and that steps toward understanding the measurement process should be formulated within that framework. This is true not only of theories where states of the measurement apparatus are given paramount importance,[1-3] but even where modifications of the

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Schrödinger equation are proposed[4] and one has the burden of showing that this leads to measurable consequences.

1.1. Brief Resumé of Earlier Work

In 1972 Hepp[5] proposed a number of models of the measurement process in the course of an article in which the infinite-volume (thermodynamic) limit was shown to play an important role in the measurement process. One of these models, now known[6] as the Hepp–Coleman model, consists of a particle passing an infinite row of noninteracting up spins, each of which it kicks over (in a slightly contrived way; the integral of the potential is restricted to a certain precise value). Because the spins do not interact, the passing particle must affect each of the apparatus spins. This model therefore does not attempt to introduce the features needed to promote the microscopic to the macroscopic. This model has been criticized by Bell[6],[3] for other reasons.

An older model by Green[8] shares with the Hepp–Coleman model the feature that the particle to be measured interacts with every degree of freedom of the apparatus. This model has the advantage, however, that special values of a potential need not be used. Green introduces metastability on a particle-by-particle basis. The x and y coordinates of each apparatus particle are maintained in disequilibrium and the presence of the system to be measured allows relaxation.

Van Kampen[9] has recently suggested that the decay of a single atom is a model of a measurement. His calculations are similar to those in the Wigner–Weisskopf description of decay and he makes use of the infinite number of degrees of freedom of the electromagnetic field to show in what way a measurement disturbs the object measured and leads to irreversibility. We believe it would be useful to elaborate his model further at both temporal ends. For the case of successful detection, the final state consists of outgoing photons. To consider this a complete measurement, we would want to see the photon absorbed and its presence amplified and registered in an apparatus that is itself macroscopic. His initial state consists of an individual atom and as such also requires some tethering to the macroscopic world. Again, the details of this tethering, for example, the residual

[3] Bell's criticism appears to rely on the construction of an observable that depends on a large number of "apparatus" variables in a precise and coherent way. The nonexistence of this kind of observable seems to us to be one of the tenets of the traditional justification for treating the density matrix as effectively diagonal. See, for example, Gottfried[7] or the discussion in ref. 2. Perhaps one could view Bell's criticism in the following way: If Hepp's analysis has indeed solved the measurement problem, then we should also learn from that analysis why it is forbidden to consider certain observables.

position uncertainty, are part of the micro–macro connection that a complete measurement model would provide.

The model that Zimanyi and Vladar[10] have proposed seems to us a great step forward, both in appreciation of the problem and in explicitness of the apparatus. They make the point that symmetry breaking is an essential feature of the measurement process. (See also ref. 11.) We would rephrase this. Amplification is the essential feature and symmetry breaking is one way to achieve amplification. (There are other ways; for example, the strong electric fields in a Geiger counter.) In any case, they provide a model where symmetry breaking does do the job. The symmetry breaking is related to the phase transition one gets from $1/r^2$ one-dimensional potentials and which was used to great advantage by Caldeira and Leggett.[12]

The $1/r^2$ potential, however, has certain drawbacks. It is weak in a variety of senses, not the least of which is the difficulty one has in proving its existence and salient properties.[13] Time correlations decay with power laws rather than with the more definitive exponentials. It seems to us that the phase transitions that play a role in practical measurements are more robust. Consider a cloud chamber. A particle passes through a container of supercooled gas, that is, a gas slightly on the metastable side of a first-order phase transition. Through a microscopic perturbation (ionization) a critical droplet forms. The liquid–gas dynamics now takes over and the droplet grows to observable size without further help from the passing particle. One has the same general principles operating that Zimanyi and Vladar require, but now in the context of a simpler and sharper phase transition.

2. THE MODEL

As indicated, our goal is to model the measurement-process amplification by means of a phase transition along the lines proposed by Zimanyi and Vladar. But we choose our model to be a caricature of a first-order transition. It turns out that the resulting calculations are fairly standard and combine techniques on phase transitions and decay that have been common knowledge for 30–50 years.

The apparatus consists of a ring of L (quantum) spins coupled to one another by ferromagnetic nearest-neighbor forces. Although L is large, the temperature T is low enough for the correlation length to be much larger than L, so that effectively the system has a first-order phase transition.[4] A magnetic field h is turned on and the spins initially arranged so that *all* of

[4] Since the correlation length is $\exp(2J/kT)$ (in our notation below), this is not a stringent requirement, even for mesoscopic L.

them are in the *unfavorable* direction with respect to this field. That is, they are in a metastable state. h is such that a single spin flip is not energetically favorable, but a double flip (of adjacent spins) is. In the language of nucleation theory, the critical droplet size is between one and two.[5]

The idea behind the operation of this apparatus is that the system to be detected (henceforth to be called the "atom") passes near the ring of spins and this passage will tend to flip one of the spins. With this spin flipped, the spontaneous flipping of one of its neighboring spins would create a larger-than-critical droplet and the entire ring of spins would subsequently turn over in a domino-like fashion, more or less inexorably and irreversibly. In this way the microscopic action of a single atom on a single spin is promoted to macroscopic dimensions. The analogy to the nucleation of a critical droplet in a bubble chamber is clear, although also clear are the differences, most notably, in our opinion, the absence of translational degrees of freedom for the atom.

Implementation of the foregoing picture requires that the spins interact with background degrees of freedom that have the combined roles of maintaining thermal equilibrium and of carrying off the energy deposited by the atom and by the exothermic nucleation process. To this end, we introduce a collection of bosons, to be called phonons, that interact with the spins. For simplicity each spin is given its own set of phonons; we have in mind that the spins are relatively far apart on the scale of the wavelength of the phonons whose coupling we use. This coupling will be taken to be weak. The phonons correspond to degrees of freedom of the substance within which the spins are embedded.

The system we measure, the "atom," will also be taken to have two states, an excited state and a ground state. The atom is transported past a particular one of the spins. If the atom is in its excited state, we will take the effect of its proximity to be a large enhancement of the coupling between that spin and its phonon bath. If the atom is in its ground state, there is no such enhancement. Thus, the measurement can be looked upon as a determination of whether or not the atom is in its excited state or, alternatively, as a detector for the excited atom.

[5] For present purposes the restriction to one dimension is unimportant. For two or three dimensions one can arrange for fields strong enough to have the critical droplet size lie between one and two spins flipped. Larger critical droplets (weaker fields) can also be considered, but then estimates would be needed for the availability of various subcritical droplets. (For one dimension we are effectively assuming T low enough for no thermal droplets to be present.) Furthermore, if one wishes to calculate the progress of droplet growth after it has exceeded critical size, this task is easy in one dimension, difficult in more than one.

A Hamiltonian embodying the features we have just described is

$$H = -J\sum_{l=1}^{L}\sigma_{zl}\sigma_{zl+1} + h\sum_{l=1}^{L}\sigma_{zl} + \sum_{k,l}\omega_k b_{lk}^{\dagger}b_{lk} + \Omega\sigma_{zA}$$
$$+\sum_{k,l}(\phi_k\sigma_{+l}b_{lk} + \phi_k^{*}\sigma_{-l}b_{lk}^{\dagger})$$
$$+\gamma(t)\sum_{k}(c_k\sigma_{+1}b_{1k}\sigma_{+A} + c_k^{*}\sigma_{-1}b_{1k}^{\dagger}\sigma_{-A}) \tag{1}$$

where

$$I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\qquad \sigma_z=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\qquad \sigma_+=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix},\qquad \sigma_-=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$
$$\sigma_{zl}=I\otimes I\otimes\cdots\otimes\sigma_z\otimes\cdots\otimes I \qquad (\sigma_z \text{ in } l\text{th position})$$
$$\sigma_{\pm l}=I\otimes I\otimes\cdots\otimes\sigma_{\pm}\otimes\cdots\otimes I \qquad (\sigma_{\pm} \text{ in } l\text{th position}) \tag{2}$$
$$J, h>0,\qquad \omega_k, \Omega\geqslant 0,\qquad \gamma(t)=\Theta(T-t)\,\Theta(t)$$

and the operators subscripted by A act on the atom's coordinates. We next relate the terms in H to the model described above in words. The coupling term $-J\sum\sigma\sigma$ is suitable for a nearest-neighbor, one-dimensional, ferromagnetic ($J>0$) quantum spin system. Modulo L addition for the subscripts of σ_{zl} corresponds to the system's being a ring. Our convention for the sign of the magnetic field term ($h\sum\sigma$) corresponds to having the spin-down state as the ground state. The free phonon term ($\sum\omega b^{\dagger}b$) describes L identical boson systems and the interaction term ($\sum\phi\sigma_+ b +$ adjoint) has each of them coupled in the same way to its particular spin. The excited state of the atom is separated by 2Ω from its ground state ($\Omega\sigma_{zA}$). Finally, the atom couples to spin number 1 only (this could be generalized[6]) and it does so for a fixed time T, our picture being that the translational degrees of freedom of the atom carry it past spin number 1. Our results should not be sensitive to the exact form of $\gamma(t)$. We leave the details of the k dependence of ω_k, ϕ_k, and c_k unspecified.

We show that with appropriate initial conditions our model evolves

[6] Spin number 1 is the one it happens to pass near. In principle, it could have come close to any of them. Introduction of the atom's position coordinates would allow our model to be generalized so as to allow, in a natural way, any of the spins to interact with the atom.

the way a measurement apparatus should. Several calculations are in order. Primarily, we calculate the transition rate for the following process:

$$\begin{pmatrix} \text{atom excited} \\ + \\ \text{all spins up} \\ + \\ \text{phonons in vacuum} \\ \text{state or nearly so} \end{pmatrix} \longrightarrow \begin{pmatrix} \text{atom in ground state} \\ + \\ \text{all spins down} \\ + \\ \text{phonons carry off} \\ \text{extra energy} \end{pmatrix} \tag{3}$$

This will be a measurement in which the measured system (the atom) is changed by the measurement. The pointer is the collective state of the L spins. One could imagine the phonons also acting as the pointer, but we will not find that necessary.

Besides the transition rate for (3), other processes and rates should be considered. Among them are (a) What is the rate for spontaneous reversal of (3) with or without the presence of the atom? (b) False positives: What is the rate for (3) in the absence of the excited atom? We perform a series of calculations to answer these and other questions.

2.1. Time Spent with Spin Down; Perturbation of the Spin-Up State

Suppose that initially all the spins are up and no phonons are excited. The atom is not present. Because of the spin–phonon coupling, the system develops an amplitude for being found with a single spin flipped; alternatively stated, the metastable state of H (in the sense of Wigner–Weisskopf theory) closest to the (all-) spin(s)-up eigenstate of the unperturbed ($\phi = 0$, $c = 0$) Hamiltonian has an admixture of spin-down amplitude. The two formulations are solved in essentially the same way.

2.2. Single-Spin Dynamics

To lowest approximation, the essential problem is that of a single spin in the background of its up-pointing neighbors. The validity of this approximation will be examined below. The reduced Hamiltonian H^1 for the single-spin dynamics is

$$H^1 = (-2J + h)\,\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sum_k (\phi_k \sigma_+ b_k + \text{adjoint}) \tag{4}$$

Our basis of quantum states is $|\pm k, k', \ldots\rangle$, the first index referring to the spin state, subsequent indices indicating the labels of excited phonons. For

doubly excited phonon modes the index will be written twice. It will be convenient not to normalize the basis so that, e.g., $|+kk'\rangle = b_k^\dagger b_{k'}^\dagger |+\rangle$, whether $k = k'$ or not.

2.3. No Phonons

The state $|-\rangle$ (no phonons) is an eigenstate of H^1 with eigenvalue $2J-h$ and, if taken as an initial condition, will remain unchanged. For this state, however, the approximation in which its neighbors remain up is not good, since the coupling term holding them there no longer exists.

2.4. Zero/One Phonons

The state $|+\rangle$ mixes with $|-k\rangle$ (all k), but no phonon states with more than one phonon enter by virtue of the form of the coupling: Dropping from $+$ to $-$ creates a phonon, but going back to $+$ destroys one. The general time-dependent wave function takes the form

$$|\psi(t)\rangle = f(t)\,|+\rangle + \sum_k g_k(t)\,|-k\rangle \tag{5}$$

Schrödinger's equation (with $\hbar = 1$) implies

$$i\dot{f} = -Wf + \sum_k \phi_k g_k \tag{6}$$

$$i\dot{g}_k = (W+\omega_k)\,g_k + \phi_k^* f \tag{7}$$

where we have defined $W \equiv 2J-h$. It is instructive first to examine the eigenvalue structure of H^1 for this (invariant) subspace. Replace $i\partial/\partial t$ by E, solve (7) for g_k, substitute in (6), and divide by f to obtain the following equation for E:

$$E + W = \sum_k \frac{|\phi_k|^2}{E-(W+\omega_k)} \tag{8}$$

Suppose there are N discrete phonon levels and ϕ is small. Take $W > 0$; this will be soon be seen to be the requirement that the critical droplet be greater than one. Graphical analysis of Eq. (8) reveals the following: There is a level with E slightly below $-W$. There are $N-1$ levels falling between successive values of $W+\omega_j$. There is one level with $E > W+\omega_N$. (We take $\omega_1 < \omega_2 < \cdots < \omega_N$.) Ultimately, irreversibility will be obtained through a continuum limit for $\{\omega_k\}$, but at this stage it is easier to work with discrete levels. In the course of deriving (8), one can also obtain the eigenfunctions

and one finds immediately that the eigenfunction for the eigenstate with $E \sim -W$ consists mainly of the state $|+\rangle$ with $O(\phi)$ corrections for the $|-k\rangle$ amplitudes. To lowest nontrivial order

$$E = -W - \sum_k \frac{|\phi_k|^2}{2W + \omega_k} \tag{9}$$

To study the time dependence of $|\psi(t)\rangle$, we use the Laplace transform $\hat{f}(s) = \int_0^\infty dt\, e^{-st} f(t)$ [and correspondingly for $\hat{g}_k(s)$], so that (7) and (6) become

$$\hat{g}_k(s) = \frac{\phi_k^* \hat{f}(s)}{is - W - \omega_k} \tag{10}$$

$$\hat{f}(s) = i\left[is + W - \sum \frac{|\phi_k|^2}{is - W - \omega_k} \right]^{-1} \equiv \frac{i}{D(s)} \tag{11}$$

where we have used the initial conditions mentioned above, $f(0) = 1$, $g_k(0) = 0$. Thus,

$$f(t) = \frac{1}{2\pi} \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} \frac{ds\, e^{st}}{D(s)} \tag{12}$$

with ε chosen to the right of all zeros of $D(s)$, which in this case [since $D(s)$ vanishes for $is \in$ Spectrum of H] means $\varepsilon =$ any positive number. There are thus two classes of contributions to f, that from the pole at $is = E_0 \sim -W$ and that from the other poles, $is > W + \omega_1$. To estimate the relative amount of time spent in $|-k\rangle$ states (for all k) we calculate the large-t behavior of $1 - |f(t)|^2$. For small ϕ, large number[7] of ω_k, and W not too small, f will be given by the first contribution alone, since the other terms will cancel against each other. The magnitude of f can thus be obtained by looking at the residue of $D^{-1}(s)$ at $-iE_0$. To lowest nontrivial order of perturbation theory (second), we write

$$D(s) = is + W - \sum |\phi_k|^2 \, [is + W - 2W - \omega_k]^{-1} \tag{13}$$

so that the residue for the pole near $is + W = 0$ is given through

$$D(s) \sim (is + W)\left[1 + \sum \frac{|\phi_k|^2}{(2W + \omega_k)^2} \right]$$

[7] Peculiarities in the time dependence can arise from subtle interplay of the $\{\omega\}$. For discussion of various such interplays, see refs. 14 and 15. The latter reference lists studies of Hamiltonians having the same general structure as our H^1.

It follows that

$$1-|f|^2 \sim \sum_k \frac{|\phi_k|^2}{(2W+\omega_k)^2} \tag{14}$$

This, incidentally, is the same answer one would get by looking at the norm squared of the coefficient of $|+\rangle$ in the $E\sim -W$ eigenfunction.

The requirement that $W=2J-h>0$ arises in doing the integral (12). For $W<0$ (actually, $W+\omega_1<0$) the root of $D(s)$ associated with a large coefficient for $|+\rangle$ is found within the spread of the ω's. We will later (for the two spin-flip case) deal with this in detail, but for now we merely mention that this means that the spin-down configuration is of lower energy even for a *single* spin flip. Then Eq. (12) describes the *decay* from spin up to spin down and going to the continuum limit, the formula $(x\pm i\varepsilon)^{-1}=P(1/x)\mp i\pi\delta(x)$ (for $\varepsilon>0$) gives the lifetime (through $i\pi\delta$) and energy shift (through P).

2.5. One/Two Phonons

For each k, the state $|+k\rangle$ mixes with all states $|-k'k''\rangle$, as well as with other $|+k'\rangle$ states. The general time-dependent state is

$$|\psi(t)\rangle=\sum_k g_k(t)\,|+k\rangle+\sum_{k,k'} h_{kk'}(t)\,|-kk'\rangle \tag{15}$$

where we make the convention for the k,k' sum that each pair (including $k=k'$) occurs just once, irrespective of order. The function $h_{kk'}$ is taken to be symmetric. Schrödinger's equation implies

$$i\dot{g}_k=(-W+\omega_k)\,g_k+\sum h_{kk'}\phi_{k'}(1+\delta_{kk'}) \tag{16}$$

$$i\dot{h}_{kk'}=(W+\omega_k+\omega_{k'})\,h_{kk'}+\phi_k^* g_{k'}+\phi_{k'}^* g_k-\delta_{kk'}\phi_k^* g_{k'} \tag{17}$$

We again perform a Laplace transform and impose the initial conditions $g_k(0)=\delta_{kk_0}$, $h_{kk'}(0)=0$. Equation (17) becomes

$$\hat{h}_{kk'}(s)=\frac{\phi_k^* g_{k'}+\phi_{k'}^* g_k-\delta_{kk'}\phi_k^* g_{k'}}{is-(W+\omega_k+\omega_{k'})} \tag{18}$$

For the one/two-phonon case there is not the simplification that occurred

for the zero/one case and instead of the immediate solution we had earlier for f, we find from (18) and the Laplace transform of (16)

$$i\delta_{kk_0} = \left[is + W - \omega_k - \sum_{k'} \frac{|\phi_{k'}|^2}{is - (W + \omega_k + \omega_{k'})} \right] \hat{g}_k(s) - \phi_k^* \sum_{k'} \frac{\phi_{k'}\, \hat{g}_{k'}(s)}{is - (W + \omega_k + \omega_{k'})} \tag{19}$$

We solve (19) by a perturbation expansion in the weak coupling ϕ. To lowest order, $g_k(t) = \delta_{kk_0}$, so that $\hat{g}_k(s) = \delta_{kk_0}/s$ to this same order. One can then obtain higher corrections to $\hat{g}_k(s)$ using Eq. (19). The energy level structure can be deduced in the same way. As before, we modify the Laplace transform equations to recover $E\psi = H\psi$ results. Let $is = E$ and drop the $i\delta_{kk_0}$ on the left-hand side. The coefficient $\hat{g}_k(s)$ is replaced by $\delta_{kk_0} + a_k$ ($a_{k_0} \equiv 0$), so that we are perturbing about the state $|+k_0\rangle$. We also adopt the notation $is = E = \varepsilon - W + \omega_{k_0}$. Equation (19) becomes

$$0 = \left[\varepsilon + \omega_{k_0} - \omega_k - \sum_{k'} \frac{|\phi_{k'}|^2}{\varepsilon - 2W + \omega_{k_0} - \omega_k - \omega_{k'}} \right] (\delta_{kk_0} + a_k) - \phi_k^* \sum_{k'} \frac{\phi_{k'}(\delta_{k'k_0} + a_{k'})}{\varepsilon - 2W + \omega_{k_0} - \omega_k - \omega_{k'}} \tag{20}$$

To lowest order, the $k = k_0$ and $k \neq k_0$ equations imply, respectively,

$$\varepsilon = -\sum \frac{|\phi_{k'}|^2 (1 + \delta_{k'k_0})}{2W + \omega_{k'}} \tag{21}$$

$$a_k = \frac{\phi_k^* \phi_{k_0}}{(2W + \omega_k)(\omega_k - \omega_{k_0})} \tag{22}$$

Equation (21) should be compared to Eq. (9). In going to a continuum limit (number of phonon modes $= N \to \infty$), ϕ scales like $1/\sqrt{N}$, while $\Delta\omega \sim \omega_{k+1} - \omega_k$ scales like $1/N$. Thus, Eq. (22) has a sensible continuum limit. In passing, we note that the Hamiltonian H^1 is bounded from below no matter how many phonons are present. This can be seen by writing

$$B_k \equiv \sqrt{\omega_k}\, b_k + \frac{1}{\sqrt{\omega_k}} \phi_k^* \sigma_-, \qquad [B_k, B_k^\dagger] = \omega_k - \frac{|\phi_k|^2}{\omega_k} \sigma_z$$

$$H^1 = \sum_k B_k^\dagger B_k - \sum_k \frac{|\phi_k|^2}{\omega_k} \sigma_+ \sigma_- - W\sigma_z$$

Thus, if $\sum |\phi_k|^2/\omega_k < \infty$, the operator H^1 will be bounded from below by virtue of the positive definiteness of $B^\dagger B$ for any B.

The amount of time spent with spin down for the one/two-phonon case is not substantially different from what it is for the zero/one case. The spin-down amplitude is basically $\hat{h}_{kk'}$, and from Eq. (18) this is seen to be $O(\phi/2W)$ (using $is \sim -W$). Squaring this and summing will give an expression very much like Eq. (14).

3. Two-Spin Dynamics; Probability of Detection

We next consider the dynamics of a pair of adjacent spins under the assumption that the two spins on either side of them are fixed and up. This will be the essential rate calculation because we are taking h large enough for the double spin flip to be an energy *drop*. Once the phonons carry away the energy of the exothermic transition, the two flipped spins will stay down. With them down, the decay of their neighbors to the down state is inevitable. We will later calculate the rate for this; we have already developed the machinery in our treatment of H^1.

As the two spins we take numbers 1 and 2. We will take $|c| \gg |\phi|$, so that during the period that the atom is nearby we will only consider the coupling of spin number 1 to its phonon bath via the c's. For number 2, the ϕ's are all there are. The duration T will be taken to be large. This calculation will later be adapted to the two-spin case using only the ϕ's, as a check on false positives. The two-spin Hamiltonian is therefore

$$\begin{aligned} H^2 = & -J\sigma_{z1}\sigma_{z2} + (-J+h)(\sigma_{z1}+\sigma_{z2}) + \sum_{l=1,2}\sum_k \omega_k b_{lk}^\dagger b_{lk} \\ & + \Omega\sigma_{zA} + \sum_k (\phi_k \sigma_{+2} b_{2k} + \phi_k^* \sigma_{-2} b_{2k}^\dagger) \\ & + \sum_k (c_k \sigma_{+1} b_{1k} \sigma_{+A} + c_k^* \sigma_{-1} b_{1k}^\dagger \sigma_{-A}) \end{aligned} \tag{23}$$

In our basis kets, labels of spin number 1 will be to the left of the semicolon, spin number 2 to the right. The atomic state will not be indicated; it is in one-to-one correspondence with the spin state of number 1. If number 1 is up, the atom is excited, etc. We will consider the initial condition $|\psi(0)\rangle = |+;+\rangle$. The general state is

$$\begin{aligned} |\psi(t)\rangle = & f(t)\,|+;+\rangle + \sum_k [g_{1k}(t)\,|-k;+\rangle + g_{2k}(t)\,|+;-k\rangle] \\ & + \sum_{k,k'} G_{kk'}(t)\,|-k;-k'\rangle \end{aligned} \tag{24}$$

where in the last sum k and k' are summed independently [in contrast to our convention in Eq. (15) for identical phonons attached to the same spin]. The time-dependent Schrödinger equation implies

$$i\dot{f} = (-3J + 2h + \Omega) f + \sum_k (c_k g_{1k} + \phi_k g_{2k}) \tag{25}$$

$$i\dot{g}_{1k} = (J - \Omega + \omega_k) g_{1k} + c_k^* f + \sum_{k'} \phi_{k'} G_{kk'} \tag{26}$$

$$i\dot{g}_{2k} = (J + \Omega + \omega_k) g_{2k} + \phi_k^* f + \sum_{k'} c_{k'} G_{k'k} \tag{27}$$

$$i\dot{G}_{kk'} = (J - 2h - \Omega + \omega_k + \omega_{k'}) G_{kk'} + g_{1k}\phi_{k'}^* + g_{2k'} c_k^* \tag{28}$$

We will calculate the time dependence of ψ, but we remark that our manipulations can also be looked upon as the finding of the imaginary part of the energy (hence the decay rate) of the metastable quantum state with energy near $-3J + 2h + \Omega$. As before, we perform a Laplace transform (recall that T is assumed large) and use the initial conditions $f(0) = 1$, $g_{1k}(0) = g_{2k}(0) = G_{kk'}(0) = 0$. From (28) we have

$$d_{kk'}(s)\, \hat{G}_{kk'}(s) = \hat{g}_{1k}(s)\, \phi_{k'}^* + \hat{g}_{2k'}(s)\, c_k^* \tag{29}$$

with

$$d_{kk'}(s) = is - J + 2h + \Omega - \omega_k - \omega_{k'} \tag{30}$$

Substitution in (26) and (27) yields

$$D_{\phi k}(s)\, \hat{g}_{1k}(s) = c_k^* \left(\hat{f}(s) + \sum_{k'} \frac{\phi_{k'}\, \hat{g}_{2k'}(s)}{d_{kk'}(s)} \right) \tag{31}$$

$$D_{ck}(s)\, \hat{g}_{2k}(s) = \phi_k^* \left(\hat{f}(s) + \sum_{k'} \frac{c_{k'}\, \hat{g}_{1k'}(s)}{d_{kk'}(s)} \right) \tag{32}$$

with

$$D_{\phi k}(s) = is - J + \Omega - \omega_k - \sum_{k'} |\phi_{k'}|^2 / d_{kk'}(s) \tag{33}$$

$$D_{ck}(s) = is - J - \Omega - \omega_k - \sum_{k'} |c_{k'}|^2 / d_{kk'}(s) \tag{34}$$

Note that $D_{\phi k}$ and D_{ck} are *not* simply related by $c \leftrightarrow \phi$, but involve different signs for Ω as well. Since, as we have seen above, a single spin flip does not lower the energy, it is clear that the decay rate will be $O(c^2\phi^2)$.

Therefore, it would be inadequate to drop the sums in (31) and (32) and substitute the result in (the Laplace transform of) Eq. (25). However, we can use (31) and (32) together to get the next level of accuracy in $\hat{g}_{lk}(s)$ by substituting the lowest order expressions back in (31) and (32). This yields

$$D_{\phi k}\,\hat{g}_{1k} = c_k^* \hat{f}\left[1 + \sum \frac{|\phi_{k'}|^2}{d_{kk'} D_{ck'}}\right] \tag{35}$$

$$D_{ck}\,\hat{g}_{2k} = \phi_k^* \hat{f}\left[1 + \sum \frac{|c_{k'}|^2}{d_{kk'} D_{\phi k'}}\right] \tag{36}$$

where we have suppressed the s argument. This implies that the Laplace transform of $f(t)$ is

$$\hat{f}(s) = i/\mathscr{D}(s) \tag{37}$$

with

$$\mathscr{D}(s) = is - (-3J + 2h + \Omega) - \sum_k \left\{ \frac{|c_k|^2}{D_{\phi k}} \left[1 + \sum_{k'} \frac{|\phi_{k'}|^2}{d_{kk'} D_{ck'}}\right] + \frac{|\phi_k|^2}{D_{ck}} \left[1 + \sum_{k'} \frac{|c_{k'}|^2}{d_{kk'} D_{\phi k'}}\right]\right\} \tag{38}$$

If the coupling were turned off, $\mathscr{D}(s)$ would vanish for $is = -3J + 2h + \Omega$ and $f(t)$ would be $\exp[-it(-3J + 2h + \Omega)]$. The coupling shifts the location of the zero and our interest is in the case in which the zero (in is) acquires an imaginary part. This will happen if one of the denominators d, D_ϕ, or D_c vanishes for positive ω. For the single-spin dynamics above, that did not happen and the real energy shift [Eq. (21)] did not lead to an irreversible transition. For the present two-spin case we will see that $d_{kk'}(s)$ can vanish, thereby providing an imaginary part in the root is and with it an exponential decay in f; that decay or transition rate is precisely the probability per unit time that the apparatus successfully notes the presence of the excited atom. For convenience we define

$$z = is - (-3J + 2h + \Omega) \tag{39}$$

and rewrite

$$d_{kk'} = z + \Delta E - (\omega_k + \omega_{k'}) \tag{40}$$

$$\begin{aligned} D_{\phi k} &= z - V_\phi - \omega_k - \sum |\phi_{k'}|^2 / d_{kk'} \\ D_{ck} &= z - V_c - \omega_k - \sum |c_{k'}|^2 / d_{kk'} \end{aligned} \tag{41}$$

with

$$\Delta E = 4(h-J)+2\Omega, \qquad V_\phi = 4J-2h-2\Omega, \qquad V_c = 4J-2h \tag{42}$$

We assume all three of the above quantities are positive. For ΔE this follows from the assumption that the critical droplet size is less than two.

In making perturbation theory arguments, we count equally powers of c or of ϕ even though we will also take $|c| \gg |\phi|$. From (38) and our earlier remarks it is clear that z is second order. It is also useful to define zeroth-order versions of the denominators

$$d^0_{kk'} = \Delta E - (\omega_k + \omega_{k'}), \qquad D^0_{\phi k} = -(V_\phi + \omega_k), \qquad D^0_{ck} = -(V_c + \omega_k) \tag{43}$$

Finally, we note that

$$\frac{1}{D_{\phi k}} = \frac{1}{D^0_{\phi k}}\left[1 - \frac{z - \sum |\phi_{k'}|^2 / d^0_{kk'}}{D^0_{\phi k}}\right] \tag{44}$$

to second order. If we rewrite Eq. (38) in terms of z, there will only be second-order and higher terms. Retaining only second and fourth orders yields

$$\begin{aligned}\mathscr{D}(s) = z\left[1 + \sum_k \left(\frac{|c_k|^2}{D^0_{\phi k}} + (c \leftrightarrow \phi)\right)\right] \\ - \sum_k \left\{\frac{|c_k|^2}{D^0_{\phi k}}\left[1 + \sum_{k'} \frac{|\phi_{k'}|^2}{d^0_{kk'}}\left(\frac{1}{D^0_{\phi k}} + \frac{1}{D^0_{ck'}}\right)\right] + (c \leftrightarrow \phi)\right\}\end{aligned} \tag{45}$$

where $(c \leftrightarrow \phi)$ means interchange c and ϕ in all appearances in the parallel expression.

As indicated above, we only wish to calculate the imaginary part of the root of $\mathscr{D}(s)$. To leading order of perturbation theory (fourth in this case), this is given by the imaginary part of the right-hand side of (45). This imaginary part is the decay[8] or transition rate Γ. Thus

$$\begin{aligned}\Gamma &= \operatorname{Im}\left\{\sum_k \frac{|c_k|^2}{-D^0_{\phi k}} \sum_{k'} \frac{|\phi_{k'}|^2}{d^0_{kk'}}\left(\frac{1}{D^0_{\phi k}} + \frac{1}{D^0_{ck'}}\right) + (c \leftrightarrow \phi)\right\} \\ &= \operatorname{Im} \sum_{k,k'} \frac{|c_k|^2 \, |\phi_{k'}|^2}{\Delta E - \omega_k - \omega_{k'}}\left(\frac{1}{V_\phi + \omega_k} + \frac{1}{V_c + \omega_{k'}}\right)^2\end{aligned} \tag{46}$$

[8] The rate Γ is associated with the amplitude. Probabilities decay with rate 2Γ.

The imaginary part comes from the singular denominator $(\Delta E - \omega_k - \omega_{k'})$. For closely spaced (essentially continuum) levels we have

$$\Gamma = \pi \sum_{k,k'} |c_k|^2 \, |\phi_{k'}|^2 \left[\frac{1}{V_\phi + \omega_k} + \frac{1}{V_c + \omega_k} \right]^2 \delta(\Delta E - \omega_k - \omega_{k'}) \qquad (47)$$

where, to avoid having to specify details of the k-space density of states, we continue to write $\sum_k$ for the integral over k.

Equation (47) is consistent with the intuitive picture of the decay that we have offered. There is a δ-function for energy conservation, requiring that the energies of the created phonons sum to ΔE (>0). The transition is also of order $|c|^2 \, |\phi|^2$. As we saw earlier, $|c|^2$ or $|\phi|^2$ represents the probability that a single spin will be down in an overall up environment. Equation (47) suggests that the transition takes place for times when *both* are down.

3.1. False Positives

The reliability of an apparatus depends on knowing the extent to which its signal is unambiguous. What is the probability of two adjacent spins flipping to the down position in the absence of an excited atom? The formula developed above, Eq. (47), will yield this quantity merely by setting $c = \phi$ and putting Ω to zero. It follows that the probability of false positives is $O(L\phi^4)$ since any pair of adjacent spins can initiate spontaneous nucleation. (The final flipped total spin state does not tell one where nucleation occurred.) Rarity of false positives then requires $L\phi^4 \ll c^2\phi^2 r$, where r is the number of excited atoms presented to the apparatus per unit time. Satisfying this condition seems to us physically reasonable, as the bubbles that we are trying to model do not involve 10^{26} particles, but rather could easily be as small as 10^6 particles. Moreover, from our theoretical point of view we only need L large enough so that questions of irreversibility and wave function position uncertainty do not arise. Even $L \sim 10^3$ should accomplish this. Moreover, a slight variation of our model could further protect against false positives. The passing atom could interact with *two* spins, so that the false positive criterion would become $L\phi^4 \ll c^4 r$. Finally, we note that Ω can be adjusted so that with the outside atom the two-spin flip exceeds criticality, whereas without it, it does not. This option for avoidance of false positives is probably physically most relevant, but because it would complicate the backreaction calculation below, we will not pursue it in detail.

3.2. Backreaction—Spin Unflipping

Suppose the proximity of the atom induces the two-spin flip. There is still no macroscopic registration of the measurement (see next section) and we must allow for the possibility that the two spins will flip up again before the neighboring spins have turned over in their "inexorable" way.

In the continuum limit the reversal does not occur. Therefore, the reversal rate necessarily depends on the detailed structure of the levels and coupling strengths. For the purposes of the present paper the continuum idealization is adequate, so the double unflipping is not a problem. Nevertheless, we will discuss briefly the mechanism of this process.

Since the unflipping discussion forces us away from the continuum approximation, it will be convenient to consider the energy eigenfunction expansion of $|\psi(t)\rangle$. By replacing $i\partial/\partial t$ by E and taking f, g_{lk}, etc., to be coefficients in a corresponding basis, Eqs. (25)–(28) become the energy-dependent Schrödinger equation. With a similar replacement [$is \to E$, drop circumflex, drop $f(0)$], the Laplace transform equations can also be used for the energy eigenfunctions. We therefore have an expansion of the form $|\psi(t)\rangle = \sum a_n |u_n\rangle \exp(-iE_n)$ with $|u_n\rangle = f^{(n)}|+;+\rangle + \sum_k g_{1k}^{(n)}|-k;+\rangle + \cdots$. The structure of the coefficients $\{f^{(n)},\ldots\}$ and $\{E_n\}$ will provide both the decay and the unflipping.

The key relation is Eq. (29). When $G_{kk'}^{(n)}$ is expressed in terms of $g_{1k}^{(n)}$ and $g_{2k}^{(n)}$ there appears a denominator $d_{kk'}$, which, as we saw in (40), is essentially $\Delta E - \omega_k - \omega_{k'}$. Since this will *nearly* vanish [recall that we are no longer in the continuum limit and the solution for E (or z) will prevent exact vanishing] $G_{kk'}^{(n)}$ can be large, despite its being reduced by a factor c or ϕ relative to $g_{lk}^{(n)}$. This means that when we express the initial state $|\psi(0)\rangle = |+;+\rangle$ as an eigenfunction expansion, the sum is *not* dominated by a single eigenfunction (as it would be if $\Delta E < 0$), but is built of many; that is, the state $|+;+\rangle$ is a poor approximation to a time-independent state. Nevertheless, we must reconcile this with the fact that it is a good metastable state with a long lifetime $1/\Gamma = O(1/c^2\phi^2)$. What is happening is that $|+;+\rangle$ is expressed as a sum over many different eigenfunctions, but that they are close in energy. Initially they are all in phase and the lifetime is basically the time necessary for them to get out of phase and have the coefficients of their $|+;+\rangle$ term incoherently cancel.

With this picture we can understand what is needed for the system to unflip. The phases must get lined up again. This is the quantum version of the Poincaré recurrence[16,17] and is related to the theory of almost periodic functions. As indicated above, we do not propose to study this problem in the present paper, but one can immediately see that the details of $\{\omega_k\}$, etc., will strongly influence the structure of the recurrences.

3.3. Registration of the Measurement; Falling Dominoes

So far we have calculated the probability of a double spin-flip. This is irreversible, but it is not macroscopic. Our next step is to show how our having initially placed the system in a metastable state—in the statistical mechanics sense—allows the micro–macro promotion.

The idea is simple. Once two spins are down, they will not come up again. Now their two immediate neighbors (numbers N and 2) are each in a background of one up spin and one down spin. For them, the nearest neighbor coupling J therefore cancels and from the standpoint of their single-spin dynamics all they feel is the field h. With h alone, their spin-*down* state has the lower energy and it is only a matter of time until they flip over and allow their phonon baths to carry off the energy. How much time? For this we can adapt our earlier single-spin dynamical calculation by setting $J=0$. The denominator we must examine is $D(s)$ of Eq. (13),

$$D(s) = is + W - \sum |\phi_k|^2 \, [is + W - 2W - \omega_k]^{-1} \tag{48}$$

Recall that W was $2J-h$, so now with J replaced by 0, W is negative. It is also convenient to define

$$z = is + W = is - h \tag{49}$$

Equation (48) becomes

$$D(s) = z - \sum \frac{|\phi_k|^2}{z + 2h - \omega_k} \tag{50}$$

As for our earlier decay calculation, we note that z is $O(|\phi|^2)$. Furthermore, and this is why setting $J=0$ makes a big change from our nondecay single-spin calculation, the denominator in the sum in (50) *does* vanish at appropriate k. As above, the decay rate γ is given by the imaginary part of D and we have

$$\gamma = \pi \sum_k |\phi_k|^2 \, \delta(2h - \omega_k) = \pi \, |\phi_{\bar{k}}|^2 \cdot (\text{density of states factor}) \tag{51}$$

where $\omega_{\bar{k}} = 2h$. The mean time for the entire ring of spins to turn over is therefore

$$L/4\gamma \tag{52}$$

This is a long time on the microscopic scale, but this fact is not a drawback to the operation of the apparatus. Once the irreversible double-spin flip has

occurred we can take our time about the registration process. Of course, if one is building a cloud chamber and wants many rapid cycles of compression and expansion, one will not want to wait too long. In this practical situation there are, however, two mitigating features. First, the "L" in (52) would become $L^{1/3}$ (where L is the number of atoms involved). Second, the critical droplets will start out somewhat larger, since the condensation will occur preferentially on relatively larger subcritical droplets.

3.4. Isolated Spin Approximation

In our first calculation above we studied single-spin dynamics, specifically the time evolution of one spin in a background $L-1$ up spins. We extended this technique by later calculating the dynamics of two spins, taking their neighbors to be up. In this section we study the validity of this isolation technique. Specifically, we examine how the conclusions of the single-spin dynamics need to be modified when looked at from the perspective of two-spin dynamics.

For single-spin dynamics, starting from an initial state $|+\rangle$, we found [Eq. (10)] the coefficient of the Laplace transform of the $|-k\rangle$ component to be

$$\hat{g}_k(s) = \frac{\phi_k^* \hat{f}(s)}{is - W - \omega_k}$$

with $W = 2J - h$. Moreover, to lowest order, the pole in is comes at $is \sim -W$, so that

$$\hat{g}_k(s) \sim \phi_k^* \hat{f}(s)/(-2W - \omega_k)$$

Let us look at the same question, allowing both spins number 1 and 2 to be dynamical (two-spin dynamics). The two-spin calculation is adapted to this problem by setting $c = \phi$ and $\Omega = 0$. From Eq. (35) we have

$$\hat{g}_{1k}(s) = \frac{\phi_k^* \hat{f}(s)}{is - J - \omega_k + O(\phi^2)} \left(1 + \sum \frac{|\phi_{k'}|^2}{d_{kk'} D_{\phi k'}}\right)$$

In this case $is \sim -3J + 2h$ so that

$$\hat{g}_{1k}(s) \sim \frac{\phi_k^* \hat{f}(s)}{-4J + 2h - \omega_k + O(\phi^2)} \left(1 + \sum \frac{|\phi|^2}{dD}\right)$$

The only substantial difference between $\hat{g}_k(s)$ and $\hat{g}_{1k}(s)$ is the factor $1 + \sum |\phi|^2/dD$. Although this term is small in a perturbation sense, it is the

place where decay enters. Thus, to the extent that decay is absent or irrelevant, the isolation technique is valid. Our calculations also implicitly involve a slight generalization of this conclusion: In our two-spin dynamics we calculate a decay rate, neglecting the fact that three-spin dynamics would give a small higher-order-three-spin decay process (apart from the two-spin dynamical decay arising from the additional nearest-neighbor pair). Because there is already decay for two spins, inclusion of the three-spin process will make no qualitative change and, as we indicated, little quantitative change. Therefore, this extended calculation has not been considered in detail. By contrast, it was necessary to go from the one-spin to the two-spin case precisely because the qualitatively significant process of decay was thereby introduced.

4. DISCUSSION

Our detector can be thought of as a small ferromagnetic ring at low temperature, with its spontaneous magnetization opposite to that of an external field. There is weak coupling between the individual spins in this magnet and the phonon bath of the substance within which this magnet is located. Spontaneous decay from the metastable state would therefore take a long time. An atom passes near one of the spins of the ferromagnet, and if the atom is in its excited state, it enhances the coupling of this spin to the phonon bath. With probability $p = (1 - e^{-2\Gamma T})$ this will induce nucleation of a critical droplet in the ring, where Γ is the decay rate calculated above [Eq. (47)] and T is the time the atom spends in the neighborhood of that spin. Once nucleated, the stable phase droplet grows to macroscopic size at a rate that is also calculated above. There is thus probability p of detection of the excited atom. The pointer for that detection is the total magnetization of the ring. The microscopic states of the two classes of pointer states (all spins up/all spins down) are nearly orthogonal, this property arising not only from the differences in spin state, but from the vastly different phonon states as well. (In our notation, the phonon bath acquires the macroscopic energy hL. One could also relate this to entropy change.)

The state of the atom at the end of the measurement has been changed –it is deexcited. In much of the literature on quantum measurement theory it has been convenient to assume that no change takes place in the measured system and to identify measurement and state preparation. That this is an idealization is discussed, for example, by Wigner.[18] This idealization is not incorporated in our model.

For the apparatus initial conditions discussed in this paper, the state of apparatus-plus-system at the end of the measurement will be a

(coherent) superposition of macroscopically distinct states. Measurement theory problems associated with the loss (for example, by "wave-packet reduction") or avoidance (because of special initial conditions) of such states are not addressed in the present paper. It is of course our intention that our model will allow these issues to be studied in a relatively concrete fashion.

In the Introduction we indicated that we sought to make a caricature of the formation of droplets in a cloud chamber. Calling our spin system a lattice gas would lead to semantic improvement, but would not provide what we feel is most seriously missing in our description: dynamical spatial coordinates. For realistic cloud chambers the temperature is not close to zero, so that a full quantum description of the initial gas state, including position coordinates, becomes problematic.

The introduction of finite temperature in our model should not present difficulty. We have shown that the properties of the 0/1-phonon state and the 1/2-phonon state are quite similar with respect to time spent (or fraction of amplitude) in the down-spin state. For finite temperature a statistical average (for example, with weight $e^{-\beta H}$) over exact microscopic apparatus states should therefore not alter our conclusions. The difference between our states and cloud chamber states is that the phase transition that we rely on persists[9] at zero temperature, allowing us to use simple microscopic apparatus states.

Besides the absence of spatial coordinates, we have another idealization deserving comment. The number of spins in our ring L is large, but for various reasons should not be of order 10^{26} (the reasons: false positives, finite temperature, and registration time). We do not consider this problem to be serious, since we can take L large enough so that microscopic reversibility and position uncertainty are no longer of concern. The flipping of our little ring magnet could then trigger other less sensitive devices and the signal amplified to the level of typewriters or cats.

Although we indicated in the Introduction that we were aware of few papers in which specific apparatus models were constructed, there is substantial literature on the general properties that an apparatus must have, density of states, ergodic properties, entropy production, etc. We mention, for example, the work of Daneri *et al.*[20] (reprinted in ref. 19). It will be interesting to see whether the model we have "constructed" conforms to these general descriptions. One feature of our model that will need elaboration for that purpose is the structure of the phonon bath, $\{\omega_k\}$, $\{\phi_k\}$, $\{c_k\}$, and density of states. In our discussion of backreactions we noted the

[9] Strictly speaking, it is not a matter of *persisting* at zero temperature; it only *occurs* at zero temperature.

importance of these properties, and many of the questions dealt with in the literature we cited have to do with irreversibility and ergodicity. It might be useful, for example, to carry the modeling process one step further and introduce the particles from which the phonon bath is abstracted.

Another use to which our model could be put is the identification of pointer states and the determination of whether those states, together with, perhaps, additional degrees of freedom, act in the ways suggested by more general considerations, for example, the inducing of superselection rules.[21] Finally, Glauber[22] has proposed a model similar in spirit (and formalism) to our single-spin restricted Hamiltonian H^1. He achieves amplification by introducing an inverted oscillator potential. This is not the same as what we have been doing. Even if one could effectively replace our background spins by an inverted parabola, there would be a small dimple on the top, representing the metastable state.

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